Practice Exam 1

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Problem 1

Let $U = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{4, 8, 12, 16, 20\}$ and $C = \{8, 10, 12, 14, 16\}$. Which of these sets is equal to $(A \cup B')' \cap C$?

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Solution

First $B' = \{2, 6, 10, 14, 18\}$,

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Solution

First $B' = \{2, 6, 10, 14, 18\}$,hence $A \cup B' = \{2, 4, 6, 8, 10, 14, 18\}$.

Problem 1

Let $U = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{4, 8, 12, 16, 20\}$ and $C = \{8, 10, 12, 14, 16\}$. Which of these sets is equal to $(A \cup B')' \cap C$?

Solution

First $B' = \{2, 6, 10, 14, 18\}$,hence $A \cup B' = \{2, 4, 6, 8, 10, 14, 18\}$.
Now, $(A \cup B')' = \{12, 16, 20\}$.

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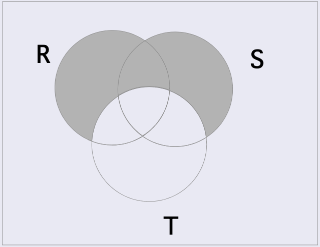
Let $U = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{4, 8, 12, 16, 20\}$ and $C = \{8, 10, 12, 14, 16\}$. Which of these sets is equal to $(A \cup B')' \cap C$?

Solution

First $B' = \{2, 6, 10, 14, 18\}$,hence $A \cup B' = \{2, 4, 6, 8, 10, 14, 18\}$.
Now, $(A \cup B')' = \{12, 16, 20\}$.
Finally, $(A \cup B')' \cap C = \{12, 16\}$.

Problem 2

Which region of the Venn diagram below is shaded?



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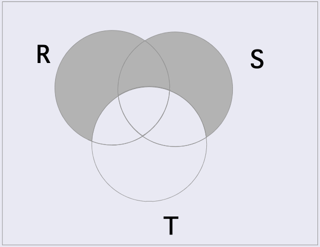

Solution

a. $(R \cup S) \cap T'$
b.$(R \cap S) \cup T'$
c. $(R \cap S) \cup T'$
d. $R \cap S$
e. $(R' \cup S')' \cap T'$

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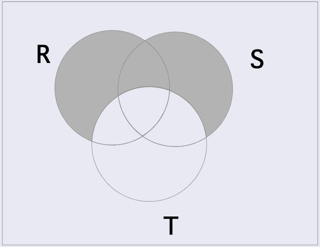

Solution

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Problem 3

R and S are subsets of a certain universal set U. If

$$n(R) = 20, \quad n(S) = 18, \quad n((S \cup R)') = 5 \quad \text{and} \quad n(U) = 35,$$

how many elements does $S \cap R$ have?

Solution

Problem 4

Out of 56 Notre Dame First Years who responded to a survey, 25 were registered in a language class, 15 in a science class, and 20 in a philosophy class. 10 were registered in both a language class and a science class, 5 in both a science and a philosophy class, and 7 in a language and a philosophy class. Three people were registered in all three. How many of the respondents were enrolled in **exactly one** of these types of classes?

Solution

Problem 5

A club consisting of ten men and twelve women decide to make a brochure to attract new members. On the cover of the brochure, they want to have a picture of two men and two women from the club. How many pictures are possible (taking into account the order in which the four people line up for the picture)?

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Solution

A1: First, choose two women for the photo from among the 12 women:

$\rightarrow C(12, 2)$ ways (I took them so that the order in which I pick them does not matter);

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A2: Then, hoose two men for the photo from among the 10 men:

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A3: Now, I order all of them:

$\rightarrow 4!$ ways;

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A3: Now, I order all of them:

$\rightarrow 4!$ ways;

This gives a total of $C(12, 2) \cdot C(10, 2) \cdot 4! = 71,280$.

Problem 6

How many four-letter words (including nonsense words) can be made from the letters of the word

EXAMINATION

if the letters that you use in the word cannot be repeated?

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There are 8 distinct letters, E, X, A, M, I, N, T, O.

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There are 8 distinct letters, E, X, A, M, I, N, T, O.

A1: Pick 4 of them (order matters)

$\rightarrow P(8,4)$ ways.

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There are 8 distinct letters, E, X, A, M, I, N, T, O.

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$\rightarrow P(8,4)$ ways.

This leads to a count of $8 \cdot 7 \cdot 6 \cdot 5$.

Problem 7

To order a pizza, you have to first choose a style (from among classic, thin crust or deep dish), a sauce (from among red, white and barbecue) and then choose toppings (from among mushroom, pepperoni, sausage, green pepper, artichoke and seaweed). If you are required to choose **at least one** topping, how many different pizzas can you create?

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To order a pizza, you have to first choose a style (from among classic, thin crust or deep dish), a sauce (from among red, white and barbecue) and then choose toppings (from among mushroom, pepperoni, sausage, green pepper, artichoke and seaweed). If you are required to choose **at least one** topping, how many different pizzas can you create?

Solution

A1: First, choose the style:

$\rightarrow C(3,1) = 3$ ways;

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Solution

A1: First, choose the style:

$\rightarrow C(3,1) = 3$ ways;

A2: Then choose the sauce:

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$\rightarrow$ (without the restriction) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$ ways;

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A3: Then choose the toppings (at least one must be chosen)

$\rightarrow$ (without the restriction) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$ ways;

$\rightarrow$ (with the restriction) $64 - 1$ ways;

So the total number of choices is $3 \cdot 3 \cdot 63 = 567$.

Problem 8

My bicycle lock uses a four-digit combination, each digit being between 0 and 9. At the moment I cannot remember the actual number, but I do remember that it either starts with a 9, or ends with 65 (in that order), or perhaps both. How many such four-digit numbers are there? (Remember that a number may start with a zero).

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Solution

Let A be the set of combinations that begin with a 9; $n(A) = 1000$ (ten choices for each of three remaining digits). Let B be the set of combinations that end with a 65; $n(B) = 100$ (ten choices for each of two initial digits). We want $n(A \cup B)$, which we know is $n(A) + n(B) - n(A \cap B)$. Since $A \cap B$ is all combinations that start with 9 and end with 65, $n(A \cap B) = 10$ (one spot to make a choice). So $n(A \cup B) = 1000 + 100 - 10$, or

$$1090$$

Problem 9

A partially eaten bag of M&M's contains 15 candies. 7 of them are red, 6 are white and 2 are blue. A sample of 4 M&M's is to be selected. How many such samples contain more blue M&M's than white?

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Solution

Case 1: 2 blues and no whites.

So, I choose 2 blues ($C(2,2)$) and then 2 red ($C(7,2)$). There are $C(2,2)C(7,2) = 21$ possibilities.

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Case 2: 2 blues and 1 white.

So, I choose 2 blues ($C(2,2)$), then 1 white ($C(6,1)$) and then 1 red ($C(7,1)$). There are $C(2,2)C(6,1)C(7,1) = 42$ possibilities.

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Case 3: 1 blues and 0 whites.

So, I choose 1 blues ($C(2,1)$) and then 3 red ($C(7,3)$). There are $C(2,1)C(7,3) = 70$ possibilities.

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So, I choose 1 blues ($C(2,1)$) and then 3 red ($C(7,3)$). There are $C(2,1)C(7,3) = 70$ possibilities.

The total number of possibilities in this either-or-or experiment is $21 + 42 + 70 = 133$.

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There are 100 Senators in the U.S. Senate, two from each of the 50 states. A committee of six Senators is to be formed, such that no two are from the same state. In how many ways can this be done?

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A1: First choose six states from which the six members will come;

$\rightarrow C(50, 6)$ ways.

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Solution

A1: First choose six states from which the six members will come;

$\rightarrow C(50, 6)$ ways.

A2: Then, for each of the six states, choose which (of 2) senators to put on the committee,

$\rightarrow 2^6$ ways.

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$\rightarrow C(50,6)$ ways.

A2: Then, for each of the six states, choose which (of 2) senators to put on the committee,

$\rightarrow 2^6$ ways.

The total count is $C(50,6) \cdot 2^6$.

Problem 11

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In how many ways can the 13 dogs be fed in the evening, one after the other, if all the dalmatians have to be fed before all the chihuahuas?

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Solution

A1: First, arrange the dalmatians in order:

$\rightarrow 4!$ (or $P(4,4)$) ways;

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Solution

A1: First, arrange the dalmatians in order:
$\rightarrow$ 4! (or $P(4,4)$) ways;

A2: Then arrange the chihuahuas in order:
$\rightarrow$ 9! (or $P(9,9)$) ways.

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In how many ways can the 13 dogs be fed in the evening, one after the other, if all the dalmatians have to be fed before all the chihuahuas?

Solution

A1: First, arrange the dalmatians in order:
$\rightarrow$ 4! (or $P(4,4)$) ways;

A2: Then arrange the chihuahuas in order:
$\rightarrow$ 9! (or $P(9,9)$) ways.

Since this is a first-then experiment, this leads to a total of $4!9! = 8709120$.

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Solution

Case 1: Family takes 3 chihuahuas:

$\rightarrow C(9,3)$ ways

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Solution

Case 1: Family takes 3 chihuahuas:

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Case 2: Family takes 3 dalmatians:

$\rightarrow C(4,3)$ ways

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Solution

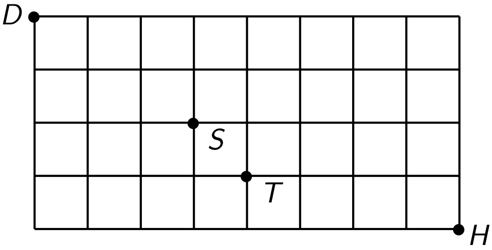
Case 1: Family takes 3 chihuahuas:

$\rightarrow C(9,3)$ ways

Case 2: Family takes 3 dalmatians:

$\rightarrow C(4,3)$ ways

Since this is an either-or experiment, this leads to a total of $C(9,3) + C(4,3) = 88$.



Problem 12

In how many ways can David walk from his house (D) to Harry's house (H), in as few blocks as possible (12)? (Ignore "S" and "T" at this point.)

Solution

Since David needs to choose which 4 of the 12 blocks he walks are south (the rest must be east), the number of ways is $C(12, 4)$ (or equivalently $C(12, 8)$).

Problem 12

Harry wants to walk from his house (H) to David's (D), but on the way he has to visit either Terri's house (T) or Sam's house (S) to pick something up (he doesn't mind if his route takes him past both S and T). In how many ways can he make this trip in as few blocks as possible (12)?

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Solution

Case 1: Harry walks from his house to David's house, via Terri's.

A1: Harry walks from (H) to (T): $\to C(5,1)$

A2: Harry walks from (T) to (D): $\to C(7,3)$

So he can do this in $C(5,1)C(7,3) = 175$ ways.

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A1: Harry walks from (H) to (T): $\rightarrow C(5, 1)$

A2: Harry walks from (T) to (D): $\rightarrow C(7, 3)$

So he can do this in $C(5, 1)C(7, 3) = 175$ ways.

Case 2: Harry walks from his house to David's house, via Sam's.

A1: Harry walks from (H) to (S): $\rightarrow C(7, 2)$

A2: Harry walks from (S) to (D): $\rightarrow C(5, 2)$

So he can do this in $C(7, 2)C(5, 2) = 210$ ways.

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Solution

Case 1: Harry walks from his house to David's house, via Terri's.

A1: Harry walks from (H) to (T): $\to C(5,1)$

A2: Harry walks from (T) to (D): $\to C(7,3)$

So he can do this in $C(5,1)C(7,3) = 175$ ways.

Case 2: Harry walks from his house to David's house, via Sam's.

A1: Harry walks from (H) to (S): $\to C(7,2)$

A2: Harry walks from (S) to (D): $\to C(5,2)$

So he can do this in $C(7,2)C(5,2) = 210$ ways.

However, we are over counting! We are counting twice the ways in which Harry can go from his house to David's, passing by both Terri's and Sam's.

Solution

Case 3: Over counting

A1: Harry walks from (H) to (S): $\rightarrow C(5,1)$

A2: Harry walks from (S) to (T): $\rightarrow 2$

A3: Harry walks from (T) to (D): $\rightarrow C(5,2)$

So he can do this in $C(5,1)2C(5,2) = 100$ ways.

Solution

Case 3: Over counting

A1: Harry walks from (H) to (S): $\rightarrow C(5,1)$

A2: Harry walks from (S) to (T): $\rightarrow 2$

A3: Harry walks from (T) to (D): $\rightarrow C(5,2)$

So he can do this in $C(5,1)2C(5,2) = 100$ ways.

We get then that there are $175 + 210 - 100 = 285$ ways of doing the stated situation.

Problem 13

Remember that a poker hand consists of a sample of 5 cards drawn from a deck of 52 cards. The deck consists of four suits (hearts, clubs, spades, diamonds), and within each suit there are 13 denominations: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king (so in total there are four cards of each denomination, one from each suit). **The order of the cards within the hand doesn't matter.**

For this problem, you may leave your answer in terms of mixtures of combinations and permutations (i.e. $C(n,r)$ and $P(n,r)$ for appropriate n and r) if you choose.

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How many poker hands are there that only include jacks, queens and kings?

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Solution

There are 12 cards in total which are jacks, queens and kings, so the number of such hands is $C(12, 5)$.

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How many poker hands are there that consist of three queens and two sevens?

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Solution

A1: First, choose 3 queens:

$\rightarrow C(4,3)$ ways;

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How many poker hands are there that consist of three queens and two sevens?

Solution

A1: First, choose 3 queens:

$\rightarrow C(4,3)$ ways;

A2: Then choose 2 sevens:

$\rightarrow C(4,2)$ ways.

Problem 13

How many poker hands are there that consist of three queens and two sevens?

Solution

A1: First, choose 3 queens:

$\rightarrow C(4,3)$ ways;

A2: Then choose 2 sevens:

$\rightarrow C(4,2)$ ways.

Therefore, the total number of ways is $C(4,3)C(4,2)$.

Problem 13

3-of-a-kind is a poker hand that includes three cards of one particular denomination and two other cards of different denominations (so three queens, one seven and one ace is an example of 3-of-a-kind, but three queens and two sevens is not). How many different 3-of-a-kind poker hands are there?

Solution

A1: First, choose the denomination which gives the triple:

$\rightarrow C(13, 1)$ ways;

Solution

A1: First, choose the denomination which gives the triple:

$\rightarrow C(13,1)$ ways;

A2: Then we choose the actual three cards:

$\rightarrow C(4,3)$ ways.

Solution

A1: First, choose the denomination which gives the triple:

$\rightarrow C(13, 1)$ ways;

A2: Then we choose the actual three cards:

$\rightarrow C(4, 3)$ ways.

A3: Next, we choose the 2 denominations which each give a single card to the five, note that the order in which we pick these two denominations does not matter:

$\rightarrow C(12, 2)$ ways;

Solution

A1: First, choose the denomination which gives the triple:

$\rightarrow C(13, 1)$ ways;

A2: Then we choose the actual three cards:

$\rightarrow C(4, 3)$ ways.

A3: Next, we choose the 2 denominations which each give a single card to the five, note that the order in which we pick these two denominations does not matter:

$\rightarrow C(12, 2)$ ways;

A4: Choose the actual fourth card (we already chose the denomination):

$\rightarrow C(4, 1)$ ways.

Solution

A1: First, choose the denomination which gives the triple:

$\rightarrow C(13, 1)$ ways;

A2: Then we choose the actual three cards:

$\rightarrow C(4, 3)$ ways.

A3: Next, we choose the 2 denominations which each give a single card to the five, note that the order in which we pick these two denominations does not matter:

$\rightarrow C(12, 2)$ ways;

A4: Choose the actual fourth card (we already chose the denomination):

$\rightarrow C(4, 1)$ ways.

A5: Choose the actual fifth card (we already chose the denomination):

$\rightarrow C(4, 1)$ ways.

Solution

A1: First, choose the denomination which gives the triple:

$\rightarrow C(13, 1)$ ways;

A2: Then we choose the actual three cards:

$\rightarrow C(4, 3)$ ways.

A3: Next, we choose the 2 denominations which each give a single card to the five, note that the order in which we pick these two denominations does not matter:

$\rightarrow C(12, 2)$ ways;

A4: Choose the actual fourth card (we already chose the denomination):

$\rightarrow C(4, 1)$ ways.

A5: Choose the actual fifth card (we already chose the denomination):

$\rightarrow C(4, 1)$ ways.

Therefore, the total number of ways is $C(13, 1)C(4, 3)C(12, 2)4 \cdot 4$.

NOTE: Many people picked the first of the two singleton denominations, and then the second, giving an answer of $13 \times \binom{4}{3} \times 12 \times 4 \times 11 \times 4$. This gives an answer that is two times too big, because it puts an unwanted order on the last two cards. For example, it counts the hand (Ace hearts, Ace clubs, A diamonds, 7 spades, 3 clubs) as different from the hand (Ace hearts, Ace clubs, A diamonds, 3 clubs, 7 spades); but these are in fact the same.

Problem 14

Six married couples are going to be in a group picture, all lined up in a row.

For each of these parts, you can give your answer using either $C(n, r)$, $P(n, r)$ or factorial notation, or you can give a numerical answer.

Problem 14

In how many ways can the 12 people line up?

Problem 14

Six married couples are going to be in a group picture, all lined up in a row.

For each of these parts, you can give your answer using either $C(n, r)$, $P(n, r)$ or factorial notation, or you can give a numerical answer.

Problem 14

In how many ways can the 12 people line up?

Solution

Twelve people need to be lined up, arbitrarily, so 12! (or $P(12, 12)$).

Problem 14

In how many ways can they line up if everyone has to be standing next to their spouse?

Problem 14

In how many ways can they line up if everyone has to be standing next to their spouse?

Solution 1

A1: Choose first person: $\rightarrow$ 12 ways;
A2: Choose second person: $\rightarrow$ 1 ways;
A3: Choose third person: $\rightarrow$ 10 ways;
A4: Choose fourth person: $\rightarrow$ 1 ways;
A5: Choose fifth person: $\rightarrow$ 8 ways;
A6: Choose sixt person: $\rightarrow$ 1 ways;

Problem 14

In how many ways can they line up if everyone has to be standing next to their spouse?

Solution 1

A1: Choose first person: $\rightarrow$ 12 ways;
A2: Choose second person: $\rightarrow$ 1 ways;
A3: Choose third person: $\rightarrow$ 10 ways;
A4: Choose fourth person: $\rightarrow$ 1 ways;
A5: Choose fifth person: $\rightarrow$ 8 ways;
A6: Choose sixt person: $\rightarrow$ 1 ways;
Keep going, to get $12 * 10 * 8 * 6 * 4 * 2$ ways of doing this.

Problem 14

In how many ways can they line up if everyone has to be standing next to their spouse?

Solution 1

A1: Choose first person: $\rightarrow$ 12 ways;

A2: Choose second person: $\rightarrow$ 1 ways;

A3: Choose third person: $\rightarrow$ 10 ways;

A4: Choose fourth person: $\rightarrow$ 1 ways;

A5: Choose fifth person: $\rightarrow$ 8 ways;

A6: Choose sixt person: $\rightarrow$ 1 ways;

Keep going, to get $12 * 10 * 8 * 6 * 4 * 2$ ways of doing this.

Solution 2

First line up the six couples in order (6! ways), then within each couple choose an order for the pair (2 choices, 6 successive times, for a total of 2^6). This gives a total of

$$6!2^6.$$

Problem 9

In how many ways can they line up if everyone has to be standing next to their spouse, with **EITHER** each husband always to the right of his wife **OR** each husband always to the left of his wife?

Solution

Case 1: Each husband to the right.

A1: Choose first person: $\rightarrow$ 6 ways;

A2: Choose second person: $\rightarrow$ 1 ways;

A3: Choose third person: $\rightarrow$ 5 ways;

A4: Choose fourth person: $\rightarrow$ 1 ways;

Keep going to get, we can do this in $6 * 5 * 4 * 3 * 2 * 1 = 6!$ ways .

Solution

Case 1: Each husband to the right.

A1: Choose first person: $\rightarrow$ 6 ways;

A2: Choose second person: $\rightarrow$ 1 ways;

A3: Choose third person: $\rightarrow$ 5 ways;

A4: Choose fourth person: $\rightarrow$ 1 ways;

Keep going to get, we can do this in $6 * 5 * 4 * 3 * 2 * 1 = 6!$ ways .

Case 2: Each wife to the right.

A1: Choose first person: $\rightarrow$ 6 ways;

A2: Choose second person: $\rightarrow$ 1 ways;

A3: Choose third person: $\rightarrow$ 5 ways;

A4: Choose fourth person: $\rightarrow$ 1 ways;

Keep going to get, we can do this in $6 * 5 * 4 * 3 * 2 * 1 = 6!$ ways .

Solution

Case 1: Each husband to the right.

A1: Choose first person: $\rightarrow$ 6 ways;

A2: Choose second person: $\rightarrow$ 1 ways;

A3: Choose third person: $\rightarrow$ 5 ways;

A4: Choose fourth person: $\rightarrow$ 1 ways;

Keep going to get, we can do this in $6 * 5 * 4 * 3 * 2 * 1 = 6!$ ways .

Case 2: Each wife to the right.

A1: Choose first person: $\rightarrow$ 6 ways;

A2: Choose second person: $\rightarrow$ 1 ways;

A3: Choose third person: $\rightarrow$ 5 ways;

A4: Choose fourth person: $\rightarrow$ 1 ways;

Keep going to get, we can do this in $6 * 5 * 4 * 3 * 2 * 1 = 6!$ ways .

So he can do this in $6! + 6! = 2 \cdot 6! \cdot$ ways.